**Problem Formulation – Linear optimization**

1. Notations:
   1. t is the index of half-hour period, t=0,1,2,…,52607
   2. d is the index of daily period, d=0,1,2,…,1095
2. Decision variables:
   1. P1\_out = The power (MW) constantly exported from battery to Market 1 during half-hour period t
   2. P2\_out = The power (MW) constantly exported from battery to Market 2 during half-hour period t
   3. P3\_out = The power (MW) constantly exported from battery to Market 3 on day d
   4. P1\_in = The power (MW) constantly imported to the battery from Market 1 during half-hour period t
   5. P2\_in = The power (MW) constantly imported to the battery from Market 2 during half-hour period t
   6. P3\_in = The power (MW) constantly imported to the battery on day d
   7. E = The energy level of the battery at the beginning of half-hour period t (MWh), where t = 0,1,2,3,…, 52608
3. Parameters used in the model:
   1. max\_charging\_rate in MW
   2. max\_discharging\_rate in MW
   3. max\_storage\_volume in MWh
   4. battery\_charging\_efficiency
   5. battery\_discharging\_efficiency
   6. prices1 = Market 1 Price [£/MWh] during half-hour period t
   7. prices2 = Market 2 Price [£/MWh] during half-hour period t
   8. prices3 = Market 3 Price [£/MWh] on day d
4. Objective function:
5. Constraints:
   1. E[0]==0, assuming the initial battery has no energy
   2. For each period t, E[t+1] == E[t] + 0.5 \* ((P1\_in[t] + P2\_in[t] + P3\_in[t//48])\*(1-battery\_charging\_efficiency) - (P1\_out[t] + P2\_out[t] + P3\_out[t//48])), the total energy in the battery at the beginning of the next period equals to the energy at the beginning of this period plus the charged energy and minus the discharged energy;
   3. For all t = 1,2,3,…,52608, E[t] <= max\_storage\_volume
   4. P1\_in[t] + P2\_in[t] + P3\_in[t//48] <= max\_charging\_rate
   5. P1\_out[t] + P2\_out[t] + P3\_out[t//48] <= max\_discharging\_rate
   6. All decision variables are non-negative
6. The code is written in python using package pulp.
7. Outputs – there are two excel files:
   1. “optimal\_profits.xlsx“ saves total yearly profits and profit for each year (£)
   2. “optimal\_decisions.xlsx” saves half-hourly battery charging/discharging (MW)